Lesson 18: The First Derivative Test

If $f'(x) > 0$, then $f(x)$ is increasing.

If $f'(x) < 0$, then $f(x)$ is decreasing.

First Derivative Test

① Find the CV's of $f(x)$ $\quad$ ($f'(c) = 0$ or DNE, and $f(c)$ is defined)

② Find where $f'(x)$ is >0 and <0.

③ If

$f(x)$

$f'(x)$: $+$ $\quad$ $-$

x: c

, then $f(c)$ is a relative max.

If

$f'(x)$: $-$ $\quad$ $+$

x: c

, then $f(c)$ is a relative min.

Otherwise, $f(c)$ is not a relative extrema.

Ex 1 $\quad y = 3x^2 + 4x$

$y' = 6x + 4 \overset{\text{set}}{=} 0$

CV: $x = \frac{-4}{6} = -\frac{2}{3}$

$\boxed{\text{rel min at } \left(-\frac{2}{3}, -\frac{4}{3}\right)}$

$f(x)$

$f'(x)$: $-$ $\quad 0 \quad$ $+$

x: $-1 \quad -\frac{2}{3} \quad 1$

rel min at

$\left(-\frac{2}{3}, 3\left(-\frac{2}{3}\right)^2 + 4\left(-\frac{2}{3}\right)\right)$

$= \left(-\frac{2}{3}, -\frac{4}{3}\right)$

Ex 2 $\quad f(x) = 3x^3 + 9x^2 - 4$

$f'(x) = 9x^2 + 18x \overset{\text{set}}{=} 0$

$9x(x+2) = 0$

$x = 0, -2$

$\boxed{\text{rel max at } (-2, 8);\ \text{rel min at } (0, -4)}$

$f'(x)$: $+ \quad 0 \quad - \quad 0 \quad +$

x: $-3 \quad -2 \quad -1 \quad 0 \quad 1$

$f(x)$

rel max

rel min

Ex 3 $\quad f'(x) = x(x+2)^2(x-3)^3$. Find relative extrema of $f(x)$.

CV's: $x = 0$, $x = -2$, $x = 3$

$\boxed{\text{rel max at } x = 0;\ \text{rel min at } x = 3}$

$f(x)$

$f'(x)$: $+ \quad 0 \quad + \quad 0 \quad - \quad 0 \quad +$

x: $-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 3 \quad 4$

rel max

rel min